Fractal speckles in diffraction regions and image plane

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ABSTRACT

Speckle patterns having fractal properties are produced in the image plane as well as in the diffraction regions of a planar diffuser. Such a pattern has no definite speckle size and exhibits a statistical self-similar feature within a limited extent. The intensity correlation function is shown to obey a power-law, which is a typical property of fractal distributions.

Keywords: Fractal speckle, doubly scattered speckle, power law, intensity correlation, diffraction

1. INTRODUCTION

Generation of random optical fields with given fractal properties is an interesting subject, since it provides a basis for applications of fractal optics.[1] For example, such fields will exhibit long tails in the spatial correlation, which may be useful in some correlation-based optical techniques. In this paper, we propose some methods for generating fractal optical fields in diffraction regions, as well as in the image plane, of a diffuser and verify them experimentally by studying statistical properties of the generated intensity distributions.

2. THEORY — FRAUNHOFER DIFFRACTION REGION

It is known that coherent light scattered in the Fraunhofer diffraction region by a random fractal object with a mass fractal property has an average intensity distribution expressed by $\langle I \rangle \propto q^{-D}$, where D and q are the fractal dimension of the object and the radial coordinate of the observation plane, respectively.[2,3]

When such a fractal object is placed in the object plane P_1 of the optical system shown in Fig. 1, the field in the Fourier transform plane P_2 has the property of $\langle I \rangle \propto q^{-D}$. Then, it can be shown analytically that, when an ordinary diffuser is placed in P_2, the intensity distribution observed in the plane P_3 has a long correlation tail obeying a power law[4]:

$$C_I(\boldsymbol{r}) = \frac{\langle I(\boldsymbol{r}')I(\boldsymbol{r}'+\boldsymbol{r})\rangle - \langle I(\boldsymbol{r}')\rangle\langle I(\boldsymbol{r}'+\boldsymbol{r})\rangle}{\langle I(\boldsymbol{r}')\rangle\langle I(\boldsymbol{r}'+\boldsymbol{r})\rangle} \propto \left(\frac{r}{MR}\right)^{2(D-2)} \quad \text{for } r \ll MR \text{ and } 1 < D < 2, \tag{1}$$

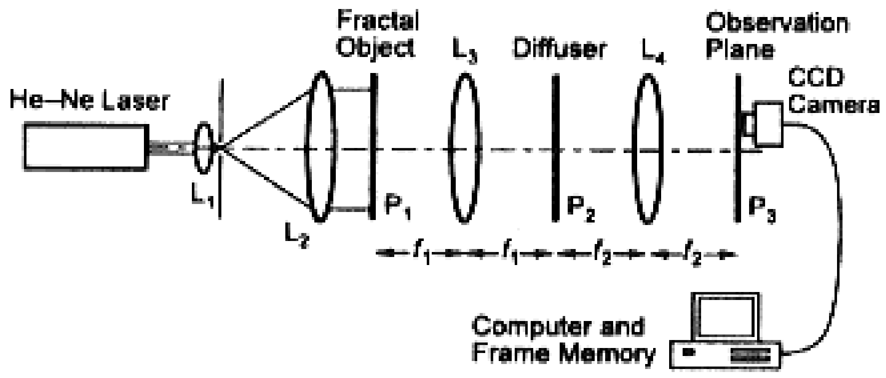

Figure 1. Experimental set-up for producing a fractal speckle pattern in the Fraunhofer diffraction region of the diffuser.

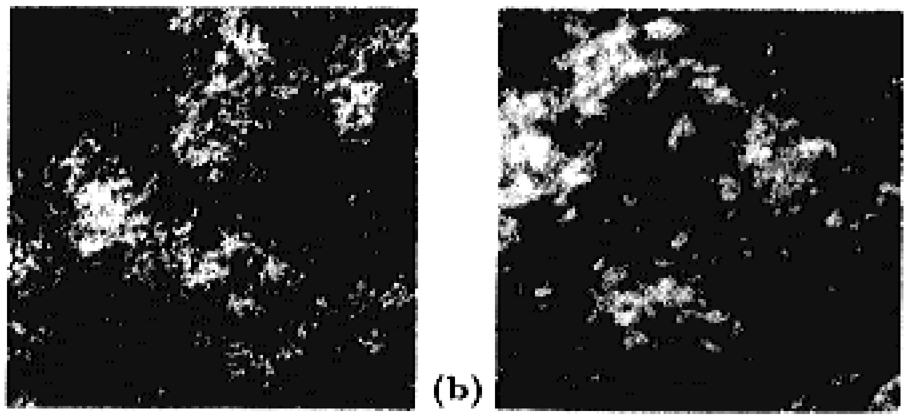

Figure 2. Statistical self-similarity of the fractal speckle produced in the Fraunhofer diffraction region of the diffuser with a fractal object of $D = 1.8$; (b) is a fourfold magnified portion of (a), having the similar appearance to (a) apart from the blur due to the finite scaling range.

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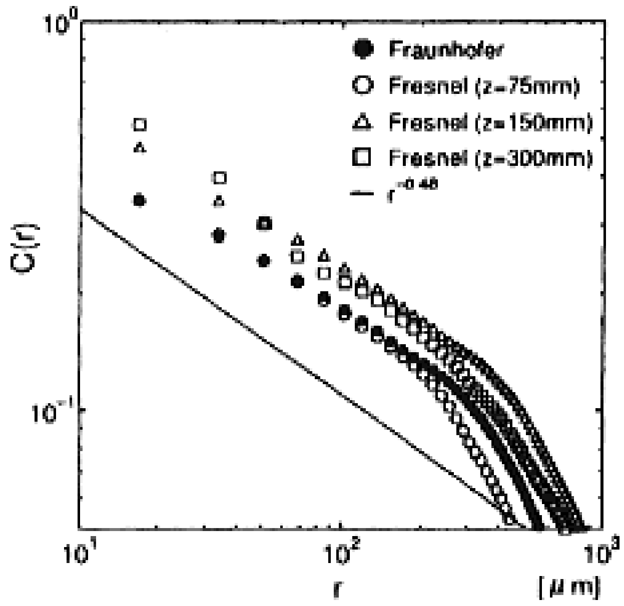

Figure 3. Intensity correlation functions of fractal speckle patterns produced in the Fraunhofer diffraction region and at three different propagation distances in the Fresnel diffraction region.

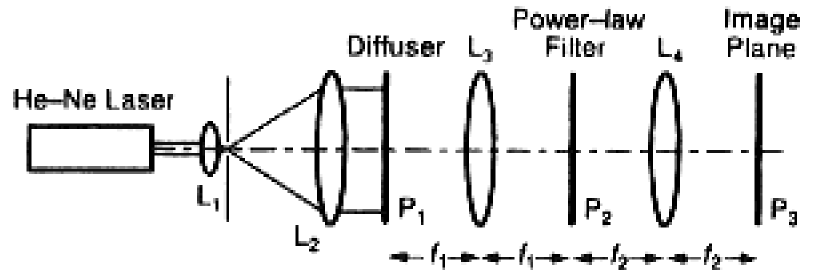

Figure 4. Experimental set-up for producing a fractal speckle pattern in the image plane of the diffuser.

where $M = f_2/f_1$, and R is a linear size of the fractal object. It is noted that the field in the plane P_3 is a kind of speckled speckles or a doubly scattered speckle pattern.[5] Equation (1) indicates that this speckle pattern has a fractal property that depends on the fractal dimension of the object. In fact, an experiment using an optical system similar to Fig. 1 gives speckle patterns that have no definite speckle size but a statistical self-similar feature as shown in Fig. 2.[6]

3. FRESNEL DIFFRACTION REGION AND IMAGE PLANE

With respect to the configuration of Fig. 1, it was also found that the fractality of the field extends over a three-dimensional region surrounding P_3,[7] a part of which is effectively the Fresnel diffraction region of the diffuser. The fractal property of speckle patterns in the Fresnel region can be examined more directly by removing the lens L_4 in Fig. 1 and by detecting speckle patterns at different propagation distances z from P_2. A result for the object with $D = 1.8$ is shown in Fig. 3, in which almost the same power law is observed for the patterns at different z as well as for the Fraunhofer region.

It is also interesting to produce the fractal speckle in the image plane of the diffuser, since some speckle-based measurements employ imaging geometries for detection. To this end, we use the configuration shown in Fig. 4, where the diffuser and an amplitude filter are placed in P_1 and P_2, respectively. In this case, the amplitude filter should have a transmittance approximately obeying the power law of q^{-D}. Since the diffuser produces a uniform speckle field in the plane P_2, the roles of the field incident on P_2 (power function) and the diffuser (quasi-white noise) in Fig. 1 are interchanged in Fig. 4, and hence, the configuration of Fig. 4 again produces the fractal speckle in P_3, which is now the image plane of the diffuser. As the power-law filter, we used the positive photographic film on which a Fraunhofer diffraction pattern of the random object was recorded.

Thus, it is concluded that fractal speckle patterns having desired power-law intensity correlations can be generated not only in the Fraunhofer diffraction region but also in the Fresnel diffraction region and in the image plane of an ordinary diffuser.

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