Nonlinear Optics, 1999, Vol. 22, pp. 229-232
Reprints available directly from the publisher
Photocopying permitted by license only

AMPLITUDE AND PHASE OF DIFFRACTED WAVE FROM MIXED HOLOGRAM

HIROFUMI FUJIWARA[a], KAZUO NAKAGAWA[a], and NOBUKATSU TAKAI[b]
[a]Muroran Institute of Technology, Muroran, 050-8585, Japan, [b]Hokkai-Gakuen University, Sapporo, 064-0926, Japan

Abstract We derive the amplitude and phase of reconstructed wave from a mixed hologram, according to a holographic coupled theory by Kogelnik. For low diffraction efficiency the amplitude is proportional to that of the object wave and the phase takes a constant value between zero and $\pi/2$.

Keywords: mixed hologram; phase of diffracted wave

INTRODUCTION

Some kinds of dye-doped-films have been used as a hologram in which an interference pattern is recorded through an irreversible photochemical change in absorption coefficient or refractive index, or both of them[1]. An interference pattern is recorded in a mixed hologram as spatial variation of both absorption coefficient and refractive index. In a pure amplitude or phase hologram, the diffracted wave has the constant phase difference of zero or $\pi/2$, compared with the phase of the reading wave, respectively. In the mixed hologram, the phase difference may take a value between zero and $\pi/2$.

In holographic or phase conjugate interferometry using a mixed hologram, this phase may be added to the intrinsic phase of the object wave. It is desirable to know beforehand the influence of the additional phase and to suppress it. We investigate the amplitude and phase of the wave reconstructed from a mixed hologram.

DERIVATION OF DIFFRACTION EFFICIENCY

We consider a mixed volume hologram of the transmission type and assume the unslanted fringes and the Bragg condition (see Fig.1). Let two coherent reference and object waves, R and O, of wavelength λ illuminate a holographic recording-material. We assume that the spatially sinusoidally modulated pattern produces corresponding photo-products inside the material. Then we consider sinusoidal variations of refractive index n' and absorption coefficient α' represented by

$$n'=\bar{n}+b\cos(Kx) \quad \text{and} \quad \alpha'=\bar{\alpha}+a\cos(Kx) \qquad (1)$$

where $K=2\pi/\Lambda=4\pi\bar{n}\sin\psi/\lambda$, the coefficients, $\bar{n}$ and $\bar{\alpha}$, are the average refractive index and absorption coefficient, and the coefficients, b and a, of sinusoidally modulated portions in n' and α' are proportional to the amplitude O.

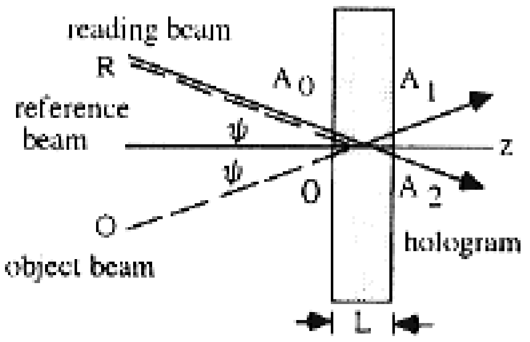

FIGURE1 Geometry for holographic record and reconstruction from a mixed volume hologram

The Kogelnik coupled wave theory [2] assumes that the reading and reconstructed waves of amplitudes A_2 and A_1 are present in the hologram. When the Bragg condition is satisfied, the following equations for A_1 and A_2 are derived as [2]

$$\frac{dA_{1,2}}{dz}+\bar{\alpha}'A_{1,2}+(a'-ib')A_{2,1}=0 \qquad (2)$$

where $\bar{\alpha}'=\alpha/\cos\psi$, $a'=a/2\cos\psi$ and $b'=\pi b/\lambda\cos\psi$. ψ being an intersecting angle between two waves O and R. It is noted that the coupling coefficient $a'-ib'$ is a complex number and is also proportional to the modulation depth of refractive index and absorption coefficient. When at z=0, $A_2(0)=A_0$ and $A_1(0)=0$, then at z=L, we have the solution

$$A_1(L)=-\xi\exp(i\theta)A_0 \qquad (3)$$

where

$$\xi = \exp(-\bar{\alpha}'L)\left[\sinh^2(a'L)+\sin^2(b'L)\right]^{1/2} \tag{4}$$

$\exp(-\bar{\alpha}'L)$ indicating a loss for the hologram, and

$$\theta = \tan^{-1}\left\{\frac{\tan(b'L)}{\tanh(a'L)}\right\} \tag{5}$$

A diffraction efficiency in amplitude representation is given by $-A_1(L)/A_0$, so that ξ and θ are the amplitude and phase of diffraction efficiency, respectively.

For a' =0 corresponding to a phase hologram with a loss, $\theta = \pi/2$. For b'=0 corresponding to an amplitude hologram with the loss, $\theta = 0$. In both cases the phases θ take constant values and does not depend on the amplitude of the object wave. On the other hand, the mixed hologram, in general, takes the phase of a value between $\pi/2$ and 0. For a'L and b'L< 0.2, tanh(a'L)~sinh(a'L)~a'L and tan(b'L)~sin(b'L)~b'L within an error of 1.5%. In this case the amplitude ξ and the phase θ are represented by

$$\xi = \exp(-\bar{\alpha}'L)\left[a'^2+b'^2\right]^{1/2}L \quad \text{and} \quad \theta = \tan^{-1}\left\{\frac{b'}{a'}\right\} \tag{6}$$

As b' and a' are proportional to the amplitude of the object wave O, ξ is proportional to the amplitude O of the object wave and θ takes a constant value which does not depend on the amplitude O. This case is acceptable for interferometric applications

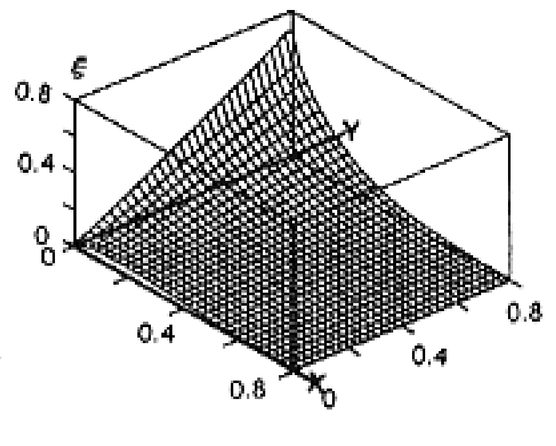

FIGURE 2 Amplitude ξ of diffraction efficiency. X=a'L, Y=b'L and $\bar{\alpha}'/a'=3$

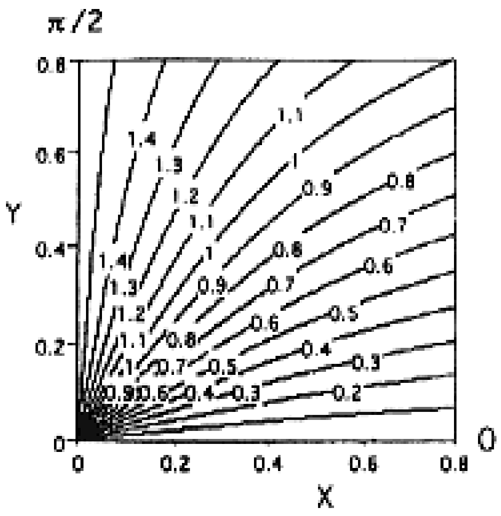

FIGURE 3 Contour map of phase θ of diffraction efficiency

using a mixed hologram because of the addition of a constant phase to the object phase, but the diffraction efficiency is low.

NUMERICAL RESULTS AND DISCUSSIONS

Figures 2 and 3 show the amplitude ξ where $\overline{\alpha}'/a'=3$ and the contour map of the phase θ of diffraction efficiency, respectively. On the Y axis (a' =0), the hologram is a pure phase one ($\theta = \pi/2$). On the X axis (b'=0), it is a pure amplitude one (θ =0). In a pure phase or amplitude hologram, the phase θ takes a constant value for large X or Y. On the other hand, for X=a'L <<1 and Y=b'L<<1 corresponding to a case of a mixed hologram, as b'/a' takes a constant value (a',b' $\neq$ 0), the mixed hologram has a constant phase θ as given by Equation(6), but the diffraction efficiency is low. Figure 3 indicates that the constant phase θ is in the region where contour curves are regarded as linear lines intersecting at the origin.

CONCLUSIONS

In a mixed hologram with low diffraction efficiency, the amplitude of the diffraction efficiency is proportional to the amplitude of the object wave and its phase takes a constant value. In holographic or phase conjugate interferometry the mixed hologram with low diffraction efficiency gives correct object information.

REFERENCES

[1.] G.C.Bjorkland, D.M.Burland, and D.C.Alvarez, *J.Chem.Phys.*, **73**, 4321(1980)
[2.] H.Kogelnik, *Bell Syst.Tech.J.*, **48**, 2909(1969)